Effects of source spatial partial coherence on temporal fade statistics of irradiance flux in free-space optical links through atmospheric turbulence

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Abstract: The temporal covariance function of irradiance-flux fluctuations for Gaussian Schell-model (GSM) beams propagating in atmospheric turbulence is theoretically formulated by making use of the method of effective beam parameters. Based on this formulation, new expressions for the root-mean-square (RMS) bandwidth of the irradiance-flux temporal spectrum due to GSM beams passing through atmospheric turbulence are derived. With the help of these expressions, the temporal fade statistics of the irradiance flux in free-space optical (FSO) communication systems, using spatially partially coherent sources, impaired by atmospheric turbulence are further calculated. Results show that with a given receiver aperture size, the use of a spatially partially coherent source can reduce both the fractional fade time and average fade duration of the received light signal; however, when atmospheric turbulence grows strong, the reduction in the fractional fade time becomes insignificant for both large and small receiver apertures and in the average fade duration turns inconsiderable for small receiver apertures. It is also illustrated that if the receiver aperture size is fixed, changing the transverse correlation length of the source from a larger value to a smaller one can reduce the average fade frequency of the received light signal only when a threshold parameter in decibels greater than the critical threshold level is specified.

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References and links
Recently, free-space optical (FSO) communications have been considered as a promising candidate for low-cost broadband access applications. However, the turbulence-induced beam scintillations can severely degrade the performance of FSO links. It has been shown that the use of a spatially partially coherent source can reduce the scintillations in FSO links through atmospheric turbulence [1,2]; on the other hand, it has also been revealed that spatial partial coherence of the source causes additional beam spreading [3], which results in an additional decrease in the average irradiance of the beam at the receiver plane. As a result, optimization for the transverse correlation length of the source is needed and has been addressed by many authors [4-6]. The temporal fade statistics of the received irradiance, e.g., the average fade frequency and average fade duration, are critical to analyze a packet FSO communication system, using forward error correction (FEC), impaired by atmospheric turbulence [7]. Thus it is interesting to understand how spatial partial coherence of the source impacts the temporal characteristics of irradiance fading of the produced beam passing through atmospheric turbulence.

Up to now, although both the average irradiance and scintillation index of beams, radiated from spatially partially coherent sources, propagating in atmospheric turbulence have been investigated in detail [1,3,8-10], the temporal characteristics of irradiance fading of these spatially partially coherent beams are rather unexplored. The Taylor frozen-turbulence hypothesis [10] has been widely used to study the temporal behavior of irradiance fluctuations induced by atmospheric turbulence. Yura and McKinley [11] developed the temporal statistics of the
turbulence-induced irradiance fluctuations in a ground-to-space FSO link. The temporal spectra of irradiance fluctuations for plane, spherical and Gaussian-beam waves in weak turbulence and for plane waves in weak-to-strong turbulence, respectively, were derived by Andrews and Phillips [10]. By taking a spatially incoherent optical field into account, Holmes et al. [12] studied the temporal spectrum of irradiance fluctuations for incoherent speckle propagation through atmospheric turbulence. Nevertheless, all these authors did not consider the case of beam waves radiated from spatially partially coherent sources. On the other hand, Xiao and Voelz [13] and Korotkova et al. [14] treated the probability density function (PDF) of irradiance fluctuations in FSO links transmitting a partially coherent beam though atmospheric turbulence and analyzed the probability of fade, described by the integral of the PDF from 0 to the threshold value, whereas their work did not involve the temporal fade statistics such as the average fade frequency and average fade duration, which depend on the root-mean-square (RMS) bandwidth of the irradiance temporal spectrum. Furthermore, practical FSO communication systems always employ receivers with a finite aperture size, and aperture averaging occurs if the aperture size in the presence of atmospheric turbulence is larger than the irradiance correlation width; in this case, it is the irradiance flux that should be taken into account rather than the irradiance. By considering a plane wave propagating in atmospheric turbulence, Andrews and Phillips [10] have shown that aperture averaging has an impact on the irradiance-flux temporal spectrum. Hence, the receiver aperture size should be taken into account when one examines the temporal fade statistics of the received light signal in a FSO link, employing a spatially partially coherent source, through atmospheric turbulence.

The purpose of this paper is to gain a deep insight into the effects of source spatial partial coherence on the temporal characteristics of irradiance-flux fading in FSO links through atmospheric turbulence. By considering a beam radiated from a spatially partially coherent source, propagating in atmospheric turbulence, and received by a finite-size aperture, we first develop the temporal covariance function of irradiance-flux fluctuations, and then derive the expressions for the RMS bandwidth of the irradiance-flux temporal spectrum. With the help of these expressions, the temporal fade statistics of the irradiance flux are calculated and analyzed for beams, radiated from sources with different transverse correlation lengths, propagating in atmospheric turbulence with various strengths.

2. Theoretical formulations

Let us consider a FSO link consisting of a transmitter, employing a spatially partially coherent source from which a beam is radiated and propagated horizontally into atmospheric turbulence, and a receiver situated at a distance $L$ from the transmitter, which uses a Gaussian lens to collect the light and focus it onto a photodetector. For mathematical tractability, here we model the beam radiated from the spatially partially coherent source as a Gaussian Schell-model (GSM) beam ([15], Sec. 5.6.4).

2.1. Temporal covariance function of irradiance-flux fluctuations

Here we develop the temporal covariance function of irradiance-flux fluctuations for GSM beams propagating in atmospheric turbulence based on the method of effective beam parameters ([16], Sec. 5.2.1). In what follows, we begin by deriving the temporal covariance function of irradiance-flux fluctuations for spatially fully coherent Gaussian beams propagating in atmospheric turbulence, and then extend it to the case of GSM beams.

Based on the derived statistical moments for beam waves propagating through complex paraxial ABCD optical systems in the presence of atmospheric turbulence ([10], p. 401, Eqs. (20) and (21)), recalling the modified Rytov theory ([16], Sec. 2.3.1) and the Taylor frozen-turbulence hypothesis for Gaussian-beam waves ([10], Sec. 8.5.3), the temporal covariance
function of on-axis irradiance-flux fluctuations at the photodetector plane for a spatially fully coherent Gaussian beam can be written as

$$B_I(\tau) = \exp[B_{Ix}(\tau) + B_{Iy}(\tau)] - 1$$  (1)

with the large- and small-scale log-irradiance-flux temporal covariance given by

$$B_{Ix}(\tau) = 8\pi^2 k^2 L \int_0^1 \int_0^\infty \kappa \Phi_n(\kappa) G_x(\kappa) J_0(\kappa \nu \tau)$$
$$\times \exp\left\{ -\frac{Lk^2}{\kappa (\Lambda_1 + \Omega_G)} \cdot \left[ \left(1 - \Theta_1 \xi \right)^2 + \Lambda_1 \Omega_G \xi^2 \right]\right\}$$
$$\times \left\{ 1 - \cos \left[ \frac{Lk^2}{\kappa} \cdot \frac{\Omega_G - \Lambda_1}{\Omega_G + \Lambda_1} \cdot \xi \left(1 - \Theta_1 \xi\right) \right]\right\} \, d\kappa \, d\xi$$  (2)

and

$$B_{Iy}(\tau) = 8\pi^2 k^2 L \int_0^1 \int_0^\infty \kappa \Phi_n(\kappa) G_y(\kappa) J_0(\kappa \nu \tau)$$
$$\times \exp\left\{ -\frac{Lk^2}{\kappa (\Lambda_1 + \Omega_G)} \cdot \left[ \left(1 - \Theta_1 \xi \right)^2 + \Lambda_1 \Omega_G \xi^2 \right]\right\}$$
$$\times \left\{ 1 - \cos \left[ \frac{Lk^2}{\kappa} \cdot \frac{\Omega_G - \Lambda_1}{\Omega_G + \Lambda_1} \cdot \xi \left(1 - \Theta_1 \xi\right) \right]\right\} \, d\kappa \, d\xi,$$  (3)

respectively, where \(k = 2\pi/\lambda\) denotes the wavenumber, \(\lambda\) is the wavelength; \(L\) is the separation distance between the transmitter and receiver; \(J_0(\cdot)\) is a Bessel function of the first kind; \(\nu\) is the average transverse wind velocity; \(\Lambda_1 = \Lambda_0 / (\Lambda_0^2 + \Theta_0^2)\), \(\Theta_1 = 1 - \Theta_1, \Theta_1 = \Theta_0 / (\Lambda_0^2 + \Theta_0^2)\), \(\Lambda_0 = 2L/(kW_G^2), \Theta_0 = 1 - L/F_0; W_0\) and \(F_0\) are the beam radius and phase front radius of curvature at the transmitter plane, respectively; \(\Omega_G = 2L/(kW_G^2)\) is the lens Fresnel parameter, \(W_G\) is the effective transmission radius of the Gaussian lens which relates to the hard aperture diameter \(D_G\) by \(D_G^2 = 8W_G^2\) ([10], p. 408); \(\Phi_n(\kappa)\) is the spatial power spectrum of refractive-index fluctuations; \(G_x(\kappa) = \text{exp}(-\kappa^2/\kappa_X^2)\) and \(G_y(\kappa) = \kappa^{11/3} \text{exp}(\Lambda_1 Lk^2 \xi^2 / k) / (\kappa^2 + \kappa_Y^2)^{11/6}\) are the large- and small-scale filter function, respectively ([10], pp. 333 and 352); \(\kappa_X\) and \(\kappa_Y\) are the large- and small-scale cutoff spatial frequency, respectively.

By introducing the Kolmogorov spectrum \(\Phi_n(\kappa) = 0.033C_n^2 \kappa^{-11/3}\) with \(C_n^2\) being the refractive-index structure constant into Eq. (2), and employing the geometrical optics approximation, one obtains

$$B_{Ix}(\tau) \cong 0.533 \sigma_R^2 \left\{ \frac{\Omega_G - \Lambda_1}{\Omega_G + \Lambda_1} \right\}^2 \int_0^1 \xi^2 \left(1 - \Theta_1 \xi\right)^2 \int_0^\infty \eta^{1/6} J_0(\omega \tau \sqrt{\eta})$$
$$\times \exp\left\{ -\frac{\eta}{\eta_X} \right\} \exp\left\{ -\frac{\eta}{\Lambda_1 + \Omega_G} \cdot \left[ \left(1 - \Theta_1 \xi\right)^2 + \Lambda_1 \Omega_G \xi^2 \right]\right\} \, d\eta \, d\xi,$$  (4)

where \(\sigma_R^2 = 1.23C_n^2 k^{7/6} L^{11/6}\) denotes the Rytov variance for a plane wave, \(\omega_0 = \nu(k/L)^{1/2}\) is the Fresnel frequency, \(\eta = Lk^2/k\), and \(\eta_X = Lk_X^2/k\). Note that, Eq. (4) reduces to Eq. (60) in Sec. 6.5.1 of [16] by letting \(\tau = 0\). By first performing the variable change \(\eta = x^2\), and then making use of the identity ([10], p. 764)

$$\int_0^\infty x^n \exp(-a x^2) J_n(bx) \, dx = \frac{b^n \Gamma\left(\frac{n+1}{2}\right)}{2^{n+1} a^{n+1} \Gamma(p+1)} \cdot _1F_1\left(p+u+1, 2; p+1; -\frac{b^2}{4a^2}\right),$$  (5)
where \( \text{Re}(\mu + p) > -1, \alpha > 0, b > 0, \) and \( {}_1F_1(\cdot) \) is a confluent hypergeometric function of the first kind, to evaluate the inside integration in Eq. (4), it follows that

\[
B_{\text{ln}x}(\tau) = 0.49\sigma_R^2 \left( \frac{\Omega_G - \Lambda_1}{\Omega_G + \Lambda_1} \right)^2 \int_0^1 \xi^2 \left( 1 - \Theta_1 \xi \right)^2 R^{7/6}(\xi) \left( \frac{7}{6}; 1; -\frac{\alpha^2\tau^2 R(\xi)}{4} \right) d\xi \quad (6)
\]

with

\[
R(\xi) = \eta_X \left\{ 1 + \frac{\eta_X}{\Lambda_1 + \Omega_G} \left[ (1 - \Theta_1 \xi)^2 + \Lambda_1 \Omega_G \xi^2 \right] \right\}^{-1} \quad (7)
\]

Along a similar line presented in Sec. 6.5.1 of [16], substituting the Kolmogorov spectrum into Eq. (3) results in

\[
B_{\text{ln}y}(\tau) \approx 1.06\sigma_R^2 \int_0^1 J_0(\omega \tau \sqrt{\eta}) \left( \frac{(1 - \Theta_1 \xi)^2}{\Lambda_1 + \Omega_G} \right)^{11/6} \exp \left\{ -\frac{(1 - \Theta_1 \xi)^2 \eta}{\Lambda_1 + \Omega_G} \right\} d\eta d\xi \quad (8)
\]

where \( \eta_y = L \xi^2 / \kappa. \) Notice that, Eq. (8) reduces to the first line of Eq. (63) in Sec. 6.5.1 of [16] if \( \tau = 0. \) By evaluating the integration over \( \xi \) in Eq. (8) and performing the variable change \( \eta = x^2, \) one finds

\[
B_{\text{ln}y}(\tau) = 1.06\pi^{1/2} \sigma_R^2 \sqrt{\frac{\Lambda_1 + \Omega_G}{\Theta_1}} \int_0^\infty \frac{J_0(\omega \tau x)}{(x^2 + \eta_y)^{11/6}} \left( \text{erf} \left( \frac{x}{\sqrt{\Lambda_1 + \Omega_G}} \right) + \text{erf} \left( \frac{x(\Theta_1 - 1)}{\sqrt{\Lambda_1 + \Omega_G}} \right) \right) dx \quad (9)
\]

where \( \text{erf}(\cdot) \) is the error function.

Equations (6) and (9) are our first theoretical contribution. By employing the aforementioned method of effective beam parameters, the large- and small-scale log-irradiance-flux temporal covariance for GSM beams can be obtained by formally replacing the conventional beam parameters \( \Lambda_1 \) and \( \Theta_1 \) in Eqs. (6) and (9) with the effective beam parameters \( \Lambda_e \) and \( \Theta_e, \) which are given by [1]

\[
\Lambda_e = \frac{\Lambda_1 N_s}{1 + 4\Lambda_1 q_e}, \quad \Theta_e = \frac{\Theta_1 + 4\Lambda_1 q_e}{1 + 4\Lambda_1 q_e} \quad (10)
\]

where \( N_s = 1 + 4q_e / \Lambda_0, \) and \( q_e = L / (2k\sigma_e^2) \) with \( \sigma_e \) being the transverse correlation length of the source. Although \( \Lambda_e \) and \( \Theta_e \) have been developed based on a random phase screen model, it has been proven that the random phase screen model is equivalent to the GSM ([10], p. 675). For GSM beams, \( \eta_X \) and \( \eta_Y \) introduced in Eqs. (4) and (8) are given by ([10], pp. 351, 352, 682)

\[
\eta_X = \left( \frac{1}{3} \frac{\Theta_e}{2} + \frac{\Theta_e^2}{5} \right)^{-6/7} \left( \frac{\sigma_\rho}{\sigma_R} \right)^{12/7} \left[ 1 + 0.56 (2 - \Theta_e) \sigma_\rho^{12/5} \right]^{-1} \quad (11)
\]

and

\[
\eta_Y = 3 \left( \frac{\sigma_R}{\sigma_B} \right)^{12/5} + 2.07 \sigma_R^{12/5} \quad (12)
\]

respectively, with

\[
\sigma_B^2 = 3.86\sigma_R^2 \text{Re} \left[ i^{5/6} {}_2F_1 \left( \frac{5}{6}, \frac{11}{6}, \frac{17}{6}; i(\Theta_e + i\Lambda_e) - \frac{11}{16} \Lambda_e^{5/6} \right) \right] \quad (13)
\]

where \( {}_2F_1(\cdot) \) is a hypergeometric function.
2.2. RMS bandwidth of irradiance-flux temporal spectrum

Although the dynamics of irradiance-flux fluctuations are completely described by their temporal spectra, generally it is the RMS bandwidth of these temporal spectra that attracts the most attention in FSO communication applications due to its importance in determining the quantities such as the average fade frequency and average fade duration of the received light signal [7,10,11].

The RMS bandwidth of the irradiance-flux temporal spectrum is defined by [7]

$$B_{\text{RMS}} = \frac{1}{2\pi} \left[ \int_{0}^{\infty} \omega^2 S_f(\omega) \, d\omega \right]^{1/2},$$  \hspace{1cm} (14)

where $\omega$ denotes the temporal angular frequency, and $S_f(\cdot)$ represents the irradiance-flux temporal spectrum. Theoretically, $S_f(\cdot)$ can be obtained by taking the Fourier transform of $B_f(\cdot)$. Nevertheless, this approach is not very tractable in our case. Thus, an alternative definition of the RMS bandwidth of the irradiance-flux temporal spectrum is employed to develop an expression for $B_{\text{RMS}}$, viz., [7]

$$B_{\text{RMS}} = \frac{1}{2\pi} \left[ -\frac{B_f''(0)}{B_f'(0)} \right]^{1/2}$$  \hspace{1cm} (15)

with $B_f''(\cdot)$ being the second derivative of $B_f(\cdot)$, which takes the form

$$B_f'(\tau) = \exp[B_{\text{inX}}(\tau) + B_{\text{inY}}(\tau)] \left\{ \left[ B_{\text{inX}}'(\tau) + B_{\text{inY}}'(\tau) \right]^2 + B_{\text{inX}}''(\tau) + B_{\text{inY}}''(\tau) \right\},$$  \hspace{1cm} (16)

where $B_{\text{inX}}'(\cdot)$ and $B_{\text{inY}}'(\cdot)$ represent the first derivatives of $B_{\text{inX}}(\cdot)$ and $B_{\text{inY}}(\cdot)$, respectively; $B_{\text{inX}}''(\cdot)$ and $B_{\text{inY}}''(\cdot)$ denote the second derivatives of $B_{\text{inX}}(\cdot)$ and $B_{\text{inY}}(\cdot)$, respectively. It needs to be pointed out that the RMS bandwidth of the irradiance-flux temporal spectrum is also referred to in the literature [10,11] as the quasi-frequency. By making use of the Leibniz rule [17,18], one finds $B_{\text{inX}}'(0) = 0$, $B_{\text{inY}}'(0) = 0$, $B_{\text{inX}}''(0) = C_1 \omega_t^2$, and $B_{\text{inY}}''(0) = C_2 \omega_t^2$ with

$$C_1 = -0.29 \sigma_R^2 \left( \frac{\Omega_G - \Lambda_e}{\Omega_G + \Lambda_e} \right)^2 \int_{0}^{\infty} \frac{x^2}{\xi^2 (1 - \Theta_e \xi)^2} R_{\xi}^{13/6} (\xi) \, d\xi, \hspace{1cm} (17)$$

$$C_2 = -0.53 \pi^{1/2} \sigma_R^2 \sqrt{\frac{\Lambda_e + \Omega_G}{\Theta_e}} \int_{0}^{\infty} \frac{x^2}{\left( x^2 + \eta^2 \right)^{11/6}} \left[ \text{erf}\left( \frac{x}{\sqrt{\Lambda_e + \Omega_G}} \right) + \text{erf}\left( \frac{x(\Theta_e - 1)}{\sqrt{\Lambda_e + \Omega_G}} \right) \right] \, dx, \hspace{1cm} (18)$$

where $R_{\xi}(\cdot)$ is obtained by replacing $\Lambda_1$ and $\Theta_1$ in Eq. (7) with $\Lambda_e$ and $\Theta_e$. Based on the results above, Eq. (15) can be written as

$$B_{\text{RMS}} = \frac{C_3 \omega_t}{2\pi}, \hspace{1cm} (19)$$

where $C_3 = \left\{ -\left( C_1 + C_2 \right) \left[ 1 + B_f^{-1}(0) \right] \right\}^{1/2}$. It is noted that $B_f(0)$ is actually the irradiance-flux variance for GSM beams, of which a good approximation has been given by [1]

$$B_f(0) = \sigma_f^2 = \exp(\sigma_{\text{inX}}^2 + \sigma_{\text{inY}}^2) - 1 \hspace{1cm} (20)$$
with
\[
\sigma_{\ln X}^2 \approx 0.49\sigma_R^2 \left( \frac{\Omega_G - \Lambda_e}{\Omega_G + \Lambda_e} \right)^2 \left( \frac{1}{3} - \frac{\eta_x}{2} + \frac{\eta_y}{5} \right) \left[ 1 + 0.4\eta_x (2 - \eta_x) / (\Lambda_e + \Omega_G) \right]^{7/6}, \tag{21}
\]

\[
\sigma_{\ln Y}^2 \approx \frac{1.27\sigma_R^2 \eta_y^{-5/6}}{1 + 0.4\eta_y / (\Lambda_e + \Omega_G)}. \tag{22}
\]

Equation (19) as our second theoretical contribution indicates that the RMS bandwidth of the irradiance-flux temporal spectrum is proportional to the Fresnel frequency \(\omega_f\) with a scaling coefficient which depends on the effective beam parameters, Rytov variance and lens Fresnel parameter. Note that \(B_{\text{RMS}}\) is in hertz. It is really difficult to develop an accurate analytical solution for the integral in Eq. (17). Hence, numerical integration is needed to evaluate it. On the contrary, upon employing the Mellin-transform-based method suggested by Sasiela ([19], Chaps. 5 and 6), the integral in Eq. (18) can be evaluated to give

\[
C_2 = -0.53\sigma_R^2 \left\{ \frac{3\pi (\Lambda_e + \Omega_G)^{1/6}}{\Theta_e \Gamma(5/6)} \right\} \left[ 2 \mathcal{F}\left[ \frac{11}{6}, \frac{1}{3}; \frac{5}{6}; \frac{\eta_y (1 - \Theta_e)^2}{\Lambda_e + \Omega_G} \right] + \frac{1 - \Theta_e}{2/3} \right] \\
\times 2 \mathcal{F}\left[ \frac{11}{6}, \frac{1}{3}; \frac{5}{6}; \frac{\eta_y (1 - \Theta_e)^2}{\Lambda_e + \Omega_G} \right] - \frac{36\eta_y^{1/6}}{s\Theta_e} \left[ 2 \mathcal{F}\left[ \frac{1}{2}, \frac{7}{6}; \frac{3}{2}; \frac{\eta_y (1 - \Theta_e)^2}{\Lambda_e + \Omega_G} \right] \right], \tag{23}
\]

where the upper sign corresponds to \(\Theta_e \leq 1\) and the lower one to \(\Theta_e > 1\); \(\mathcal{F}(\cdot)\) is a generalized hypergeometric function.

2.3. Temporal fade statistics

The temporal fade statistics of the received light signal, such as the fractional fade time, average fade frequency and average fade duration, are of great interest in FSO communication systems using on-off-keying (OOK) modulation combined with direct detection in the presence of atmospheric turbulence. These statistics are used, in this paper, as measures to characterize the temporal characteristics of irradiance-flux fading in FSO links through atmospheric turbulence. Note that, although the turbulence-induced fading of the received light signal is sensed with the help of a photodetector, to avoid the mathematical complications associated with the detection noise ([20], Sec. 7.2.3), here we only concentrate on the irradiance-flux fading caused by atmospheric turbulence.

By assuming that the irradiance-flux fluctuations are an ergodic process, the fractional fade time, defined as the percentage of time when the irradiance flux is below a given threshold \(I_{\text{th}}\), equals the probability of fade given by [21]

\[
p_{\text{fade}}(\mu_{\text{th}}) = \int_{0}^{\mu_{\text{th}}} p_I(\mu) \, d\mu, \tag{24}
\]

where \(p_I(\mu) = 2(\alpha^2 + \beta^2 + 2)^{1/2} I_{\alpha - \beta} [2(\alpha^2 + \beta^2 + 2)^{1/2}] / [\Gamma(\alpha)\Gamma(\beta)]\) is the PDF of irradiance-flux fluctuations under weak-to-strong turbulence conditions [10,16,21]. \(\Gamma(\cdot)\) denotes the gamma function, \(I_{\alpha - \beta}(\cdot)\) represents a modified Bessel function of the second kind, \(\alpha = 1 / [\exp(\sigma_{\ln X}^2) - 1], \beta = 1 / [\exp(\sigma_{\ln Y}^2) - 1]\), the normalized irradiance flux \(\mu = I / \langle I \rangle\).
with $I$ being the irradiance flux and $\langle I \rangle$ the average irradiance flux, and the normalized threshold $\mu_{th} = I_{th}/\langle I \rangle$. Equation (24) can be written as an analytical expression involving two generalized hypergeometric functions ([10], p. 452). However, numerical errors may occur under certain conditions when this analytical expression is evaluated by current software programs. Hence, in this paper, the integral appearing in Eq. (24) is numerically evaluated. It should be noted that the fractional fade time does not depend on the Fresnel frequency.

The average fade frequency, defined as the expected number per unit time of negative crossings of the irradiance flux below a given threshold $I_{th}$, is written by ([10], p. 456)

$$\langle f_{fade}(\mu_{th}) \rangle = 2\sqrt{\frac{2\pi}{\alpha\beta}} \sigma_{RMS} \Gamma(\alpha) \Gamma(\beta) (\alpha\beta\mu_{th})^{(\alpha+\beta-1)/2} K_{\alpha-\beta} \left(2\sqrt{\alpha\beta\mu_{th}}\right).$$

(25)

The average fade duration characterizes the average time at which the irradiance flux stays below a given threshold $I_{th}$ and is defined by ([10], p. 456)

$$\langle t_{fade}(\mu_{th}) \rangle = \frac{p_{fade}(\mu_{th})}{\langle f_{fade}(\mu_{th}) \rangle}.$$ 

(26)

3. Numerical calculations and analysis

In this section, we examine the effects of source spatial partial coherence on the temporal characteristics of irradiance-flux fading in FSO links using a receiver with a finite-size aperture through atmospheric turbulence. The basic parameters used in the calculations are as follows: $\lambda = 1550$ nm, $W_0 = 2.5$ cm, $F_0 = \infty$, and $L = 2$ km. To account for weak-to-strong atmospheric turbulence, the Rytov variance $\sigma^2_R$, used as a measure of turbulence strength, is specified as 0.3, 1 and 10, corresponding to weak, moderate and strong atmospheric turbulence, respectively. The parameter $W_G$ is assumed to be $10^{-3}$, 0.1 and 1 cm. $W_G = 10^{-3}$ cm is used to consider the case of a receiver with a very small aperture size, which nearly behaves like a “point receiver.”

The RMS bandwidth of the irradiance-flux temporal spectrum, i.e., the quasi-frequency, is very important for accurately determining the average fade frequency and average fade duration. Hence, it is worthwhile to first provide some insights into the effects of source spatial partial coherence on the RMS bandwidth of the irradiance-flux temporal spectrum. Figure 1 illustrates the scaled RMS bandwidth $B_{RMS}/\omega_t$ of the irradiance-flux temporal spectrum as a function of the relative transverse correlation length $\sigma_c/W_0$ with various combinations of $\sigma^2_R$.

![Fig. 1. RMS bandwidth of the irradiance-flux temporal spectrum, scaled by the Fresnel frequency $\omega_t$, as a function of $\sigma_c/W_0$.](image)

The RMS bandwidth of the irradiance-flux temporal spectrum, i.e., the quasi-frequency, is very important for accurately determining the average fade frequency and average fade duration. Hence, it is worthwhile to first provide some insights into the effects of source spatial partial coherence on the RMS bandwidth of the irradiance-flux temporal spectrum. Figure 1 illustrates the scaled RMS bandwidth $B_{RMS}/\omega_t$ of the irradiance-flux temporal spectrum as a function of the relative transverse correlation length $\sigma_c/W_0$ with various combinations of $\sigma^2_R$. 
Fig. 2. Fractional fade time in terms of the threshold parameter in decibels. (a) $\sigma_R^2 = 0.3$; (b) $\sigma_R^2 = 1$; (c) $\sigma_R^2 = 10$.

and $W_G$. Notice that, the Fresnel frequency $\omega_t$ depends on neither $C_n^2$ nor $W_G$. As a result, the differences between the curves in Fig. 1 are only caused by the distinctions in their associated parameters concerning the turbulence strength and effective transmission radius of the Gaussian lens. It is seen from Fig. 1 that with the same values of $\sigma_R^2$ and $\sigma_c/W_0$, a greater $W_G$ results in a smaller $B_{RMS}/\omega_t$. This fact manifests that aperture averaging can reduce the RMS bandwidth of the irradiance-flux temporal spectrum. It is observed from Fig. 1 that for all curves, although the scaled RMS bandwidth only exhibits very slight variations when $\sigma_c/W_0 < 10^{-1}$ or $\sigma_c/W_0 > 10^1$, there exists an observable increase in its value as $\sigma_c/W_0$ reduces from $10^1$ to $10^{-1}$. This phenomenon suggests that decreasing the transverse correlation length of the source can lead to an increase in the RMS bandwidth of the irradiance-flux temporal spectrum. Furthermore, it is found from Fig. 1 that in the case of $W_G = 10^{-3}$ or $0.1$ cm, with the same value of $\sigma_c/W_0$, stronger atmospheric turbulence leads to a larger $B_{RMS}/\omega_t$ regardless of what value of $\sigma_c/W_0$ is considered, and in the case of $W_G = 1$ cm, however, $B_{RMS}/\omega_t$ with a relatively large value of $\sigma_c/W_0$, e.g., $\sigma_c/W_0 = 10^2$, decreases as the turbulence strength increases. Hence, the effects of turbulence strength on the RMS bandwidth of the irradiance-flux temporal spectrum depend on both the receiver aperture size and the transverse correlation length of the source.

Now we investigate the effects of source spatial partial coherence on the temporal fade statistics of the irradiance flux. According to the analysis above, we choose four typical values of the transverse correlation length as $\sigma_c = 0.05W_0$, $0.5W_0$, $10W_0$ and $\infty$; the first three values of $\sigma_c$ correspond to spatially partially coherent sources, and the last one denotes a spatially fully coherent source. Figure 2 shows the fractional fade time in terms of the threshold parameter in
decibels, \( F_T = 10 \log_{10}(\mu_{th}^{-1}) \), with different combinations of \( W_G \) and \( \sigma_c \). It is found from Fig. 2 that both source spatial partial coherence and aperture averaging can affect the fractional fade time under all conditions of turbulence strength, i.e., a larger receiver aperture size or smaller transverse correlation length leads to a lower fractional fade time. The reason is qualitatively explained as follows. Physically, on the one hand, spatial partial coherence of the source can smear the scintillation pattern at the receiver plane and hence fill many of the fade regions; on the other hand, aperture averaging can average multiple independent intensity patches over the aperture area and thus reduce the probability of fade. Moreover, it is observed from Figs. 2(a) – 2(c) that the curves associated with the same \( W_G \) and various \( \sigma_c \) become closer to each other as atmospheric turbulence grows stronger. This fact means that the reduction in the fractional fade time due to a decrease in the transverse correlation length of the source becomes less significant in stronger atmospheric turbulence. Nevertheless, it is illustrated by Fig. 2(c) that even in strong atmospheric turbulence with \( \sigma_R^2 = 10 \), aperture averaging can still reduce the fractional fade time substantially. Finally, it is noted that with the same \( W_G \), an indiscernible difference between the curves for \( \sigma_c = 10 W_0 \) and those for \( \sigma_c = \infty \) is shown by Fig. 2. This implies that as far as our particular numerical example is concerned, for a fixed \( W_G \), a partially coherent source with \( \sigma_c = 10 W_0 \) nearly leads to the same fractional fade time as a fully coherent one with \( \sigma_c = \infty \). Physically, if \( \sigma_c/W_0 \gg 1 \), the global coherence parameter [22] of the nominal partially coherent source approaches 1, and hence it almost behaves like a fully coherent source.

Figure 3 illustrates the scaled average fade frequency as a function of the threshold parameter \( F_T \) with different combinations of \( W_G \) and \( \sigma_c \). Notice that, as in Fig. 1, the average fade frequency is

\[
\langle f_{\text{av}}(10^{-\mu_{th}}) \rangle \psi
\]

Threshold parameter \( F_T \) (dB)

(a)

(b)

(c)

\[
\langle f_{\text{av}}(10^{-\mu_{th}}) \rangle \psi
\]

Threshold parameter \( F_T \) (dB)

\[
\begin{align*}
\text{dash} & : W_G = 10^{-3} \text{cm}, \sigma_c = 0.05 W_0 \\
\text{dot} & : W_G = 10^{-3} \text{cm}, \sigma_c = 0.5 W_0 \\
\text{dash-dotted} & : W_G = 10^{-3} \text{cm}, \sigma_c = 10 W_0 \\
\text{circle} & : W_G = 10^{-3} \text{cm}, \sigma_c = \infty \\
\text{dash} & : W_G = 1 \text{cm}, \sigma_c = 0.05 W_0 \\
\text{dot} & : W_G = 1 \text{cm}, \sigma_c = 0.5 W_0 \\
\text{dash-dotted} & : W_G = 1 \text{cm}, \sigma_c = 10 W_0 \\
\text{circle} & : W_G = 1 \text{cm}, \sigma_c = \infty
\end{align*}
\]
Fig. 4. Average fade duration, multiplied by the Fresnel frequency $\omega_t$, as a function of the threshold parameter in decibels. (a) $\sigma_R^2 = 0.3$; (b) $\sigma_R^2 = 1$; (c) $\sigma_R^2 = 10$.

frequency in Fig. 3 is scaled by the Fresnel frequency, which is independent of the transverse correlation length, turbulence strength, threshold parameter and effective transmission radius of the Gaussian lens. It is observed from Figs. 3(a) and 3(b) that any two curves associated with the same $W_G$ intersect with each other within the range $0 < F_T < 10$. For description convenience, here we refer to the specific threshold level in decibels associated with the aforementioned intersection as the critical threshold level $F_c$ for a pair of curves associated with the same $W_G$. By comparing each pair of curves in Figs. 3(a) and 3(b) associated with the same $W_G$ and different $\sigma_c$, one finds that a smaller $\sigma_c$ leads to a larger average fade frequency when $F_T < F_c$ and a lower one when $F_T > F_c$. This fact results in a statement that for two spatially partially coherent sources with different $\sigma_c$, if the receiver aperture size is fixed, the source with the smaller $\sigma_c$ can lead to a lower average fade frequency only when the normalized threshold $\mu_{th}$ is specified as a value smaller than $10^{-0.1F_c}$. It is also found from Fig. 3(c) that any two curves associated with $W_G = 10^{-3}$ cm also intersect with each other within the range $0 < F_T < 10$, but those associated with $W_G = 1$ cm do not. At a first glance, it would appear that the curves associated with $W_G = 1$ cm in Fig. 3(c) contradict the said statement. However, this seeming contradiction is resolved if we extend the range of the $F_T$-axis from 10 to a much larger value; namely, when $F_T$ grows further more beyond 10, in the case of $W_G = 1$ cm, the aforementioned intersection actually occurs too, although this is not displayed in Fig. 3(c). By comparing the curves in Figs. 3(a) – 3(c) associated with different $\sigma_R^2$, it is found that with a fixed receiver aperture size, the critical threshold level $F_c$ associated with a given pair of curves with different $\sigma_c$ increases as the turbulence strength enhances. Unlike the source spatial partial coherence, one finds from Figs. 3(a) – 3(c) that aperture averaging can always reduce the average fade frequency significantly.
under all conditions of turbulence strength no matter what value of the threshold parameter $F_T$ is specified. Furthermore, it is interesting to be noted that in the case of strong atmospheric turbulence with $\sigma^2_R = 10$, the dB threshold level at which the average fade frequency reaches its maximum is obviously greater than 0, i.e., the level below which the received irradiance flux most often crosses is smaller than the average irradiance flux. Once again, we note that for a fixed $W_G$, the curves associated with $\sigma_c = 10W_0$ and $\infty$ almost merge together.

The scaled average fade duration in terms of the threshold parameter $F_T$ with different combinations of $W_G$ and $\sigma_c$ is shown in Fig. 4. By comparing the curves in Figs. 4(a) – 4(c), it is found that although the average fade duration reduces monotonously as $F_T$ rises, i.e., as $\mu_{\text{th}}$ decreases, under all conditions of turbulence strength, the reduction in the average fade duration with increasing $F_T$ becomes slightly less significant as atmospheric turbulence grows stronger. In addition, it needs to see from Fig. 4 that with the same $W_G$, a smaller transverse correlation length of the source always leads to a shorter average fade duration. This behavior is different from that of the average fade frequency in Fig. 3, i.e., source spatial partial coherence can reduce the average fade duration no matter what value of the threshold parameter is considered. However, one also finds from Fig. 4(c) that the reduction in the average fade duration caused by decreasing the transverse correlation length becomes very inconsiderable in strong atmospheric turbulence for the relatively small receiver aperture with $W_G = 10^{-3}$ cm. Thus, a statement can be made that source spatial partial coherence becomes nearly ineffective in reducing the average fade duration in strong atmospheric turbulence for a FSO communication system using a relatively small receiver aperture. It needs to be noted from Fig. 4 that a larger receiver aperture size results in a longer average fade duration. This is due to the reason that enlarging the receiver aperture size leads to a more remarkable decrease in the average fade frequency than that in the fractional fade time. Finally, as before, we find from Fig. 4 that with a fixed $W_G$, the average fade duration for $\sigma_c = 10W_0$ can not be clearly differentiated from that for $\sigma_c = \infty$.

4. Conclusions

In this paper, novel expressions for the temporal covariance function of irradiance-flux fluctuations of GSM beams propagating in weak-to-strong atmospheric turbulence have been derived by making use of the method of effective beam parameters. According to the obtained expressions, by employing the Leibniz rule, formulae for the RMS bandwidth of the irradiance-flux temporal spectrum due to GSM beams passing through weak-to-strong atmospheric turbulence have been further developed. Based on the achieved theoretical results, the temporal fade statistics of the received light signal in FSO links, using spatially partially coherent sources, through atmospheric turbulence can be easily determined, and hence the effects of source spatial partial coherence on the temporal fade statistics of the received light signal can be examined and analyzed.

It has been found that reducing the transverse correlation length of the source can induce an increase in the RMS bandwidth of the irradiance-flux temporal spectrum. With a fixed receiver aperture size, a smaller transverse correlation length of the source results in a lower fractional fade time and shorter average fade duration; however, when atmospheric turbulence grows strong, source spatial partial coherence becomes less effective in decreasing the fractional fade time for the case of both large and small receiver apertures, and in reducing the average fade duration for the case of small receiver apertures. On the other hand, for a FSO link with a given receiver aperture size, a reduction in the average fade frequency, caused by changing the transverse correlation length of the source from a larger value to a smaller one, can occur only when the threshold parameter in decibels is specified as a value greater than the critical threshold level.

The research work in this paper provides a better understanding of how source spatial partial coherence can influence the temporal fade behavior of FSO communications, which is critical for the design and optimization of FSO systems.
coherence impacts the temporal fade statistics of the received light signal in FSO communication systems, employing spatially partially coherent sources, in the presence of atmospheric turbulence. Our results are useful in analyzing and designing a packet FSO communication system impaired by atmospheric turbulence.

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